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Spatial Training Improves Children's Mathematics Ability

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We tested whether mental rotation training improved math performance in 6- to 8-year-olds. Children were pretested on a range of number and math skills. Then one group received a single session of mental rotation training using an object completion task that had previously improved spatial ability in children this age (Ehrlich, Levine, & Goldin-Meadow, 2006). The remaining children completed crossword puzzles instead. Children's posttest scores revealed that those in the spatial training group improved significantly on calculation problems. In contrast, children in the control group did not improve on any math tasks. Further analyses revealed that the spatial training group's improvement was largely due to better performance on missing term problems (e.g., $4 + \_\_\_\_ = 11$).

Previous research has established a link between spatial ability and mathematics—children and adults who perform better on spatial tasks also perform better on tests of mathematical ability (Burnett, Lane, & Dratt, 1979; M. B. Casey, Nuttall, & Pezaris, 2001; Delgado & Prieto, 2004; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Holmes, Adams, & Hamilton, 2008; Kyttälä, Aunio, Lehto, Van Luit, & Hautamaki, 2003; Lubinski & Benbow, 1992; McKenzie, Bull, & Gray, 2003; McLean & Hitch, 1999; Rasmussen & Bisanz, 2005). This link may be based on shared underlying processes. Brain imaging studies confirm that similar areas are activated when people process both spatial and number tasks (see Hubbard, Piazza, Pinel, & Dehaene, 2005, and Umiltà, Priftis, & Zorzi, 2009, for reviews). There also is behavioral evidence that the two are connected. For example, studies indicate that number is mentally represented in several spatial formats (e.g., the Spatial Numerical Association of Response Codes (SNARC) effect, object files, etc.; see Mix & Cheng, 2012, for a review). The connection between space and math is so compelling that many now believe spatial training could be an important resource for improving performance in Science, Technology, Engineering, and Mathematics (STEM) disciplines (Lubinski, 2010; Newcombe, 2010; Uttal et al., 2012). In fact, the National Council of Teachers of Mathematics (2010) now recommends integrating spatial reasoning into the elementary mathematics curriculum. However, these recommendations may be premature as there is not yet direct evidence that spatial training can improve math learning. In the present study, we report what may be the first such evidence.

The Connection Between Spatial Ability and Math

Many studies have demonstrated that people who are better at spatial tasks also excel in mathematics. Although most of this research has been conducted with teens and adults, there is

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enough evidence in young children to suggest a link that could be leveraged by educators. For example, strong visuospatial working memory is related to superior performance on counting tasks (Kyttälä et al., 2003), number line estimation (Geary et al., 2007), and nonverbal problem solving (Rasmussen & Bisanz, 2005), as well as better overall math performance (Alloway & Passolunghi, 2011; Gathercole & Pickering, 2000; Meyer, Salimpoor, Wu, Geary, & Menon, 2010; Raghubar, Barnes, & Hecht, 2010). Studies also have found that performance on mental rotation tasks, such as the Block Design subtest of the Wechsler Intelligence Scale for Children-Third Edition, is significantly correlated with composite scores of math achievement throughout school age, from kindergarten to 12th grade (Johnson, 1998; Lachance & Mazzocco, 2006; Markey, 2010; Mazzocco & Myers, 2003). It is important to know that space and math are related in the early grades, because many studies indicate that early intervention is critical for closing achievement gaps in math (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Saxe, 1987; Starkey, Klein, & Wakeley, 2004).

Additional evidence that space and math are related comes from research on spatio-quantitative representations, such as the mental number line and object files (Dehaene, Bossini, & Giraux, 1993; Kahneman, Treisman, & Gibbs, 1992; Noles, Scholl, & Mitroff, 2005; Siegler & Opfer, 2003; Trick & Pylyshyn, 1994). There is excellent evidence, for example, that people represent quantitative magnitudes in terms of space as a mental number line starting in early childhood and continuing into adulthood. One indication is that people are faster to identify small numbers with their left hand than they are with their right hand (and vice versa), suggesting that they represent quantities on a linear number line with their own bodies at the midpoint (i.e., the SNARC effect; Berch, Foley, Hill, & Ryan, 1999; Dehaene et al.; de Hevia & Spelke, 2009; Fias, 2001; Fischer, 2003; Lourenco & Longo, 2009; van Galen & Reitsma, 2008). Another indication is that people represent small quantities using a spatial tracking process. It has long been recognized that people immediately apprehend small numbers (i.e., 1 through 4) without counting (Jensen, Reese, & Reese, 1950; Jevons, 1871; Kaufman, Lord, Reese, & Volkmann, 1949; Taves, 1941). More recent research has revealed these rapid number estimates are generated by a spatial individuation process that uses pointers to track object locations (Kahneman et al.; Noles et al.; Trick & Pylyshyn). Finally, the conventions for written mathematics rely heavily on spatial relations, and both adults and children are sensitive to these relations. For example, adults performed worse at solving algebraic equations when the distances among terms were manipulated (e.g., $2 + 3 \times 4$ vs. $2+3 \times 4$; Fischer, Borchert, & Bassok, 2011; Landy & Goldstone, 2007). Perhaps for related reasons, McNeil and Alibali (2004) reported that fourth graders struggle to solve math equations in the form $4 + 3 + 5 = 4 + \_\_\_$ even though they readily solve standard forms of the same problem (e.g., $4 + 3 + 5 = \_\_\_$). Indeed, extreme deficits in visual-perceptual skills are indicative of a particular math learning disability (Geary, 1993; Rourke, 1993).

In summary, the existing literature provides a firm basis for concluding that spatial ability and math share cognitive processes beginning early in development. Correlational studies confirm that spatial ability is related to math ability throughout development, including the early elementary grades. Research also indicates that quantities are represented in spatial formats (i.e., the mental number line and object files) beginning in early childhood and persisting into adulthood. Finally, spatial ability is required to understand mathematical symbols. Taken together, there is excellent reason to hypothesize that spatial training would improve math learning.

Can Spatial Ability Be Improved through Training?

A variety of training approaches have led to improved spatial ability. This finding lends support to the idea that spatial training can improve mathematics performance in the same way as spatial ability itself can be trained (Baenninger & Newcombe, 1989; Ehrlich, Levine, & Goldin-Meadow, 2006; Heil, Rosler, Link, & Bajric, 1998; Hsi, Linn, & Bell, 1997; Kail, 1986; Newcombe & Frick, 2010; Sorby & Baartmans, 2000; Uttal et al., 2012; Vasta, Knott, & Gaze, 1996). What remains controversial is whether this improvement leads to gains beyond better performance on the training items.

Although some studies claim to have demonstrated transfer from training to novel items or tasks (e.g., DeLisi & Cammarano, 1996; Terlecki, Newcombe, & Little, 2008; Wallace & Hofelich, 1992), many have failed to obtain such evidence (B. M. Casey et al., 2008; Kail, 1986; Kail & Park, 1990; Morgan, Bartram, & Clarke, 1984). For example, Kail and Park found that whereas 11-year-olds could be trained to recognize alphanumeric symbols in various orientations, children did not show similar improvement on untrained items. Findings like these led some to conclude that existing spatial training effects are quite specific and context-bound (National Research Council, 2006; Wright, Thompson, Ganis, Newcombe, & Kosslyn, 2008). However, in a recent meta-analysis of the spatial training literature, Uttal et al. (2012) reached very different conclusions. Specifically, they found no differences in the magnitude of training effects whether a study tested near or medium transfer (effect sizes = .47 and .49, respectively). Unfortunately, there were not enough cases of far transfer to determine whether effect sizes for those studies were comparable, but these findings at least suggest that spatial training transfers beyond the training task.

All that said, it is not clear that spatial training would need to transfer to other spatial tasks to have an impact on math. There may well be productive connections between spatial training and math, even if these do not transfer to other spatial tasks, because the transfer could occur at a very specific process level. In fact, it is possible that certain spatial tasks are more similar to certain math tasks than they are similar to other spatial tasks if the same processes are engaged.

Can Spatial Training Improve Math Performance?

Although the idea that spatial training might improve math learning is not new (Bishop, 1980; Smith, 1964), surprisingly few studies have actually tested it. The most closely related research has demonstrated that experience with spatio-quantitative materials (e.g., walking along a number line mat, free play with blocks, or experience with board games/video games) leads to improvement in math (Fischer et al., 2011; Graziano, Peterson, & Shaw, 1999; Ramani & Siegler, 2008; Wolfgang, Stannard, & Jones, 2001). For example, Ramani and Siegler found that experience playing the board game Chutes and Ladders led to more accurate placement of numbers on a number line.

These studies are encouraging because they suggest that math learning is sensitive to spatial input. However, none has tested whether training on spatial cognition per se (e.g., mental rotation, visuospatial working memory, etc.) leads to gains in math, per se (e.g., calculation). Instead, this work tends to combine space and math in both the training and the tests. For example, Chutes and Ladders involves moving a number of spaces (as indicated by a spinner) along a linear path that is marked with numerals. Thus, it is a spatial task but also has a strong quantitative component. Similarly, the outcome measure (placement on a number line) has both spatial and quantitative components. In contrast, the present study provides spatial training using a mental rotation task that has no obvious quantitative components and then tests its effects on a mathematical task (calculation) with no obvious spatial components. This provides a more direct test of the hypothesis that improved spatial ability will cascade into improved mathematical ability.

METHOD

Participants

Fifty-eight children participated ($M_{age} = 7;1$, range $= 6;1–8;5$). An additional 6 children were recruited but excluded because they performed above 75% on the math pretest. We targeted 6- to 8-year-olds because basic calculation skills are developing but are not mastered in this age range. Also, previous research has established that by this age, mental rotation ability and math performance are related (Kyttälä et al., 2003) and training can improve mental rotation ability (Ehrlich et al., 2006). Children were randomly assigned to either the spatial training group ($n = 31$) or a no-training control group ($n = 27$). Participants were drawn from a diverse, but predominantly Caucasian middle-class, population in Michigan. There were 17 boys in the spatial training group and 17 boys in the control group.

Materials and Procedure

Children first completed three pretests (two spatial tests and one math test). On a different day (scheduled within 1 week of the first), they completed one 40-minute training session followed immediately by the three posttests.

For children in the spatial training condition, the session consisted of mental rotation practice. We used a mental rotation task shown to be trainable in previous research with 6- to 8-year-olds (Ehrlich et al., 2006; see Figure 1). In this task, children see two parts of a flat shape and then point to one of four pictures that shows the shape as a whole. As feedback, children were given the two parts on separate pieces of cardstock and asked to verify or change their choices after moving them together, thus creating the whole. Children in the control condition completed crossword puzzles similar to those used as filler tasks in previous research on spatial ability (Cherney, 2008; Rizzo et al., 1999).

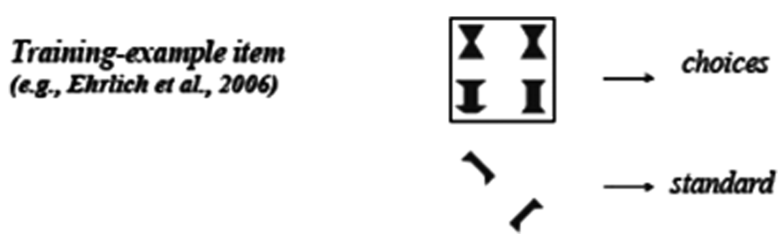

FIGURE 1 Training example item (e.g., Ehrlich et al., 2006).

The three tests were:

Mental rotation test. This test consisted of 16 novel trials exactly like those used in the mental rotation training task, except that the shapes were printed on the bottom of a single sheet of paper rather than on movable pieces. Children responded by circling the resulting shape from among four choices at the top of each page. This test measured whether our training procedure was adequate to cause improvement on the same mental rotation task.

Spatial relations subtest (test of primary mental abilities [PMA]). To see whether our spatial training led to general improvement in spatial ability, we also gave children the Spatial Relations subtest from the Test of Primary Mental Abilities (Thurstone, 1974). This test consists of 27 items in which children choose from among four incomplete figures the one that will combine with the standard to make a square. Children received 4 familiarization items followed by the 27 test items. A 6-minute time limit was imposed.

Math test. We tested the effects of spatial training on math performance with a set of 27 addition and subtraction problems. Items included single-digit number fact problems (e.g., $4 + 5 = \_\_\_\_$), two- and three-digit calculation problems (e.g., $56 + 6 = \_\_\_\_$; $124 + 224 = \_\_\_$), and missing-term problems (e.g., $4 + \_\_\_\_ = 12$). The Cronbach's alpha interitem reliability coefficient for this test was .92.

RESULTS

To determine whether the spatial training group outperformed the control group on any of the outcome measures and also to avoid Type 1 error, we first conducted a multivariate analysis of covariance with children's three posttest scores as dependent measures and their pretest scores as covariates. The analysis indicated a significant difference favoring the spatial training group (Wilks's $\lambda = .64$), $F(3, 51) = 9.64$, $p < .001$, $\eta^2 = .36$;[1] see Table 1). Univariate tests indicated that this difference was evident on both the Mental Rotation Test, $F(1, 55) = 16.23$, $p < .001$, $\eta^2 = .23$, and the math test, $F(1, 55) = 8.73$, $p = .005$, $\eta^2 = .14$. There was no significant group difference on the Spatial Relations subtest, however, suggesting that our mental rotation training did not lead to a general improvement in spatial ability.

To understand the significant group differences on the math test, we carried out separate analyses of covariance for each of the specific problem types (number fact problems, multidigit calculation, and missing-terms problems). There was a significant difference favoring the spatial training group on missing-term problems (e.g., $2 + \_\_\_\_ = 7$ or $9 - \_\_\_\_ = 5$), $F(1, 55) = 7.80$, $p = .007$, $\eta^2 = .12$. However, no significant differences between conditions emerged for either the number-fact problems or multidigit calculations (see Table 2). A slightly different pattern was revealed using paired-sample t-tests (one-tailed) to compare children's pretest and posttest scores. For spatial-training children, there was significant improvement on missing-term problems, $t(30) = 2.79$, $p = .005$, and multidigit calculations, $t(30) = 1.65$, $p = .05$, but not on number-fact problems, $t(30) = 0.36$, $p = .36$. In contrast, children in the control group failed

[1]We estimated effect size using eta squared (η^2), which is appropriate for use with analyses of covariance. For this measure, .02 is considered a small effect, .13 is considered a medium effect, and .26 is considered a large effect (Bakeman, 2005; Cohen, 1988).

TABLE 1
Means and Standard Deviations (*SDs*) of Pretests and Posttests by Condition

	Spatial training group		*Control group*	
Tests	*Pretest (SD)*	*Posttest (SD)*	*Pretest (SD)*	*Posttest (SD)*
Mental Rotation Test	11.35 (3.42)	14.55* (2.12)	11.96 (2.39)	12.70 (2.71)
Spatial Relations Subtest (PMA)	10.26 (4.67)	11.74 (4.34)	12.33 (5.16)	13.04 (4.81)
Math Test	11.10 (6.71)	12.45* (7.61)	10.9 (5.39)	10.6 (6.49)

*$p < .01$.

TABLE 2
Means and Standard Deviations (*SDs*) of Subsets of Math Subskills by Condition

	Spatial training group		*Control group*	
Tests	*Pretest (SD)*	*Posttest (SD)*	*Pretest (SD)*	*Posttest (SD)*
Missing-term problems	3.68 (3.11)	4.52* (3.56)	4.15 (3.08)	3.90 (3.13)
Number-fact problems	4.25 (1.41)	4.32 (1.45)	4.30 (1.10)	4.22 (1.28)
Multidigit calculation	3.16 (2.89)	3.61 (3.06)	2.44 (2.06)	2.56 (2.69)

*$p < .01$.

to show significant improvement on any of the math subskills: missing term, $t(26) = 1.19$, $p = .13$; multidigit, $t(26) = 0.49$, $p = .32$; number fact, $t(26) = .34$, $p = .37$.

DISCUSSION

Although previous studies have demonstrated that learning math with spatial tools can lead to improvement in quantitative tasks (e.g., Fischer et al., 2011; Ramani & Siegler, 2008), our study is the first to show a direct effect of spatial training per se on math performance in early elementary-aged children. We found that even a single session of spatial training led to significant improvement on certain problems. This result adds further support to claims that spatial cognition and mathematical reasoning are connected, but it is unique in that it is the only study to demonstrate a causal link.

It is interesting that the spatial-training effect was strongest on missing-term problems. Previous research has shown that children have an inflexible understanding of the equal sign and prefer to solve equations in a familiar, left-to-right order (Knuth, Stephens, McNeil, & Alibali, 2006; McNeil & Alibali, 2005). Perhaps our results reflect children's attempts to solve missing-terms problems by mentally rotating missing-term equations into a more conventional format (e.g., $2 + \_\_\_ = 7$ becomes $\_\_\_\_ = 7 - 2$, or $9 - \_\_\_ = 5$ becomes $\_\_\_ = 9 - 5$). If so, our brief mental rotation practice may have facilitated or primed this underlying process, rather than having led to deep conceptual change. Nonetheless, our findings are indicative of shared cognitive processing that is sensitive to input, thus raising the possibility that more extensive training would lead to more pervasive changes.

Another possible mechanism by which mental rotation training improved children's math performance could be increasing visuospatial working memory (VSWM) capacity. Recall that children with better VSWM also exhibit better math performance (Alloway & Passolunghi, 2011; Gathercole & Pickering, 2000; Geary et al., 2007; Kyttälä et al., 2003; Meyer et al., 2010; Raghubar et al., 2010; Rasmussen & Bisanz, 2005). Perhaps mental rotation training improved children's VSWM, which, in turn, supported better calculation performance. If so, it is interesting that children's spatial improvement did not transfer to performance on the Spatial Relations subtest (PMA). After all, improvements in VSWM should lead to very broad improvements in both space and math, but our effects were relatively narrow—appearing mainly on missing-term problems. Still, there was improvement on the calculation test as a whole and on both missing-term and multidigit problems. Also, any interpretations must be tempered by the fact that our spatial training was very brief. It is possible more extensive improvements would be observed with additional training.

It also would be interesting to see if the same patterns are evident given different kinds of spatial training. We chose a training task that had been successful in previous work aimed at improving spatial ability in young children, but there were many alternatives. For example, we did not find an effect of mental rotation training on place-value concepts, but perhaps such effects would be obtained with visuospatial perception training (e.g., figure matching) if visuospatial perception shares processes with place-value notation whereas mental rotation does not.

Further research is clearly needed to completely understand the nature of these effects—the critical variables that mediate these training effects and a full description of the links between specific spatial skills and specific math skills. However, the present findings are important because they provide at least an existence proof that spatial training can improve math performance. This suggests there is great instructional potential in further exploration of the causal relations between spatial cognition and mathematics.

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